The Research Behind Maths Pathway

Justin Matthys, BSc MSc (Phys.) Dip. Ed., Co-founder of Maths Pathway

Introduction

Ability spread can prevent teaching for conceptual understanding

The Australian Curriculum places a heavy emphasis on “conceptual understanding” in mathematics. Students should be reasoning mathematically, know the links between components of mathematics, and be able to apply their understanding creatively (Australian Curriculum Assessment and Reporting Authority, 2015a). This clearly goes far beyond mere “procedural understanding”, where students memorise operations without necessarily understanding their underlying meanings (Arslan, 2010).

In the past, the tendency in school-based mathematics education has been to emphasise procedural understanding heavily. Many students have been taught step-by-step to memorise maths recipes by rote. Indeed, the most common challenges encountered by students exactly match what one would expect from rote learning: forgetting things from year to year; being unable to transfer skills into other subject areas; being unable to problem-solve; only feeling confident with ready-made recipes; and calling maths one of the ‘hardest’ subjects (Barr, Doyle, Clifford, De Leo, & Dubeau, 2003).

There is a good reason for this common teaching practice: conceptual understanding is notoriously difficult to build. To obtain this deep understanding, students must actively construct their mathematical knowledge from existing knowledge (Cobb, 2010). This in turn requires a hierarchical learning paradigm wherein each piece of new learning has strict pre-requisites that must first be met (Jones & Russell, 2006). In other words, students can only conceptually understand a new piece of maths if they have already mastered everything that leads to that point.

Therefore, if there is a *range* of student starting-points in a class, the teacher is presented with a seemingly impossible situation. For any new piece of maths, there are at best only a few students who are actually ready to build conceptual understanding; it is likely that most students are missing at least one key pre-requisite. The teacher may then be forced to fall back on procedural instruction: when learning a step-by-step recipe, all students are able to access the lesson.

This challenge is common in Australia, because our classes have students with a wide range of levels and therefore, learning needs (Farr, 2010, pp. 26-40). To help narrow the spread in any given class, many schools “stream” students into different classes based on their perceived ability (Forgasz , 2010). Streaming is at best a partial solution. Controversy surrounds the practice of streaming, noted in over 300 studies as having “minimal effects on learning outcomes and profound negative equity effects” (Hattie, 2009). Moreover, even after streaming the spread can still be considerable: teachers can face almost the same barriers to building conceptual understanding in streamed classes as they do in classes of mixed ability.

What is the solution?

Differentiated instruction as the most elegant solution

Happily, the Australian Curriculum leaves room for a solution: it does not dictate precisely what students should be learning based solely on their age. Instead, the "age-appropriate" work is used as a starting point for planning, after which the teacher can tailor the student's learning program to their own specific learning needs. This includes selecting related learning outcomes from levels below or above the student's age – and allows for different students learning different things at the same time (Australian Curriculum Assessment and Reporting Authority, 2015b). The benefits include "alleviating" or "eliminating" disengagement, promoting learning which is accessible for all, as "different students learn in different ways", and research has proven this approach immensely "benefits all learners to work at their appropriate level" often excelling through this method of instruction (Morgan, 2013).

The reporting structures in Australian states and territories also allows for a fully differentiated learning program. For instance, Victorian teachers are required to report on the entry-level and exit-level of students against different content strands (Department of Education and Training, 2013). If a student enters at a particular level, and exits at a higher level, this demonstrates growth against a continuum. Interestingly, this does *not* require a single lock-step course that is identical for all students; students can still working to the same set of standards, but encounter different parts of the curriculum at different times.

A highly idealised model of differentiated instruction – where every student's learning needs are precisely met all the time – is often talked about but seldom realised. Almost without exception, Australian secondary maths teachers take their classes through a lock-step course: there are standard assessments at the end of every unit, and common learning outcomes for the whole class. The differentiated learning that does happen is generally restricted within this model. For instance, easy, medium or hard versions of the same mathematical concept can be given to the low, medium and high level student in the class each lesson. This is certainly better than no differentiation at all, but does not go nearly far enough to address the challenges of building conceptual understanding.

So how can a teacher with a multi-ability class overcome the barriers to conceptual understanding? The solution is deceptively simple: each student is at a different point of readiness, so this requires each student to be learning different things at the same time. This model looks simple because it is easy to state, but implementation is another story entirely.

There are two main challenges with implementing this model. The first is purely practical: for a single teacher to manage this for a whole class requires almost super-human ability: precise, up-to-date data on the particular learning needs of each individual; ability to provide 25 different learning activities simultaneously to 25 different students; and ongoing assessment that is different for each student.

The second challenge is pedagogical. Even removing a key barrier to conceptual understanding is no guarantee that students will learn successfully; it could be possible to introduce more problems than are solved. Individualised work has its place, but it is also vital to preserve the best elements of whole-class instruction including direct instruction, student discourse and collaborative group-work.

So far as we are aware, Maths Pathway is the first organisation in the world to overcome both of these challenges. Since July 2013, hundreds of Australian primary and secondary teachers have adopted the Maths Pathway pedagogical model, supported by a custom eLearning system and professional development. This has proved to be completely practicable and manageable, and moreover is enabling outstanding classroom pedagogy.

What follows is a brief discussion first of how Maths Pathway allows teachers to overcomes the practical challenges via its classroom workflow, followed by a description of the pedagogical elements that combine to make this workflow extremely effective.

Maths Pathway as a practical tool

Part 1 – set up and diagnosis

When Maths Pathway is used in classrooms, teachers and students log into separate web applications. The teacher application is used initially to set up class lists and timetables. Teachers can also input unit planning data by specifying what units of work will run over the semester. Each unit is given a name, a time-period over which to run, and a specified 'target point' which is a set of content descriptions selected from the Australian Curriculum.

At the start of each unit, students complete an adaptive diagnostic assessment online. This assessment (and the subsequent work within the unit) is restricted to the target specified by the teacher as well as any pre-requisite skills in the curriculum at least as far back as grade 1 level. The diagnostic identifies precisely where each student has gaps and competencies within that subset of the curriculum, and automatically designs a personal learning plan for each student based on that data.

Part 2 – coursework

After the diagnostic is completed, the class embarks on a two-week work cycle with a formal assessment at the end. Students view and manage their set work using the computer. This is an ordered set of learning activities, drawn from the bank of resources Maths Pathway provides. Different work is automatically assigned to students with different learning needs.

The primary form the learning activities take are hand-written tasks students complete in exercise books. Students view questions and worked solutions on their computer screens, and have an element of choice in which questions to answer in which way. The expectation for students is to correct their own work as they go through, but also to use the worked solutions if they are stuck on particular problems. The activities are written in a constructivist manner: students are able to acquire new knowledge by completing carefully scaffolded problems.

Other material supplements the primary written work. Videos attached to each activity provide direct instruction for students who may require it. Many activities also have virtual manipulatives attached, which often help students to build or play with a visual model of a mathematical concept.

When a student feels they are finished with an activity, they indicate on the computer that they are ready to be assessed on it, then move directly onto the next assigned activity. Once all assigned activities for the fortnight are finished, students complete some guided test revision and then require the teacher's permission to access extra work. Teachers control how much work is assigned to individuals each fortnight, and are also able to track the completion of this course work using the teacher web application.

Depending on the age of the student, 50% to 80% of class time is taken up with this individualised work, which is often set for homework as well. The remainder of the time is spent with the class learning in a hands-on fashion, solving problems in groups, or participating in class discussion about key mathematical concepts.

Part 3 – assessment

At the end of the fortnight, students sit a formal maths test. The teacher is able to control the timing of the test, but generally aims for the last lesson in the fortnight. Because students have been working on different things, each test looks different. This is generated automatically by the Maths Pathway system in two sections: one section for students to complete online which is marked automatically; and one printed section for students to complete by hand which is marked by the teacher. This latter section is emailed to the teacher automatically the previous day as a single pdf which can be sent to the printer double-sided with every page pre-labelled. The printed section contains only those questions which it would be inappropriate to assess online: questions that call for written explanations, drawings or working out.

Under test conditions, the teacher hands out the printed part of the test. Students also log into their web application and are prompted to enter the unique code on their printed test to access the online portion. Students then complete both the online and offline questions in any order, and have the opportunity to proof-read their answers before submitting.

Every practical contingency here is allowed for. If students finish early, new learning activities appear automatically for them so they can continue working without pause. If students require extra time, the teacher can temporarily suspend their access to the online part of the test. If students run out of time, the teacher can permanently stop their online test. If a student is absent, they are able to sit the test in the following lesson, or else skip that assessment entirely and are assessed on four weeks' work at once at the end of the following fortnight.

Part 4 – feedback

Once a student's test is submitted, the teacher uses their web application to help mark the printed portion. Every test is different, but teachers are automatically provided with a solution set unique to each student. Teachers provide written feedback on the tests, and enter the marks into the online system.

In the lesson after the test, teachers return the marked written sections to students. Students are able to again enter their test's unique access code in order to begin an interactive self-reflection process. This allows them to see the results of the online and offline sections as a cohesive whole. Students see how each correct and incorrect response aligns to the activities they completed in the previous fortnight.

Each activity the students complete corresponds to a set of questions on their test. Students earn 'mastery' of that activity when they answer every question in the set correctly. If they made only minor errors that they themselves, there is an opportunity to correct this during the reflection process. Any large errors or omissions students make in their tests trigger a repeat of the learning activity as part of the next fortnight's work; such activities are re-tested at the end of the next fortnight using different questions.

Once students have completed their test reflections, they move straight onto the next fortnight's work. This is a combination of new learning activities they have unlocked by mastering pre-requisites, and old learning activities they are re-doing because they failed to obtain mastery the first time. Data-tracking and teacher intervention prevent students from becoming stuck on a particular activity indefinitely.

In the lesson or two at the start of the fortnight, teachers also call students up one at a time to conduct personal feedback interviews using a special online interface. This contains all the information about the previous fortnight, with particular focus on the question-by-question performance on the test. Using this conversation prop, teachers provide targeted, personal verbal feedback to each student in the class. This feedback focuses on the choices the student made during the test and also during the fortnight leading to the test.

Part 5 – data and reporting

Students and teachers have access to various metrics that track student performance. All metrics are tracked over time, and averages are also looked at. At least one data point is collected for each metric each fortnight.

The chief metric is "growth rate". For students, this is the main measure of success. This is a percentage figure related to how many activities the student demonstrated mastery in over a given time period. If the student is on track to grow by 1 grade-level each year, this displays as 100%. If the student is on track to grow by 2 grade levels each year, this displays as 200%. This is not derived from completion of work, but rather from demonstration of mastery via fortnightly assessments.

Other metrics include:

- Overall level: the grade level of the student overall, taking account of all gaps and competencies through the curriculum. This is equivalent to an Australian Curriculum grade level with a high evidence threshold; for a student to be reported at Level 10A they would need to have shown mastery over every single part of the curriculum.
- Grade level by strand/substrand: the same as overall level above, but broken down into different topics in the curriculum.
- Accuracy: the proportion of work attempted that students demonstrate mastery over in their assessments, shown as a percentage.
- Effort: the proportion of work assigned that students completed within each fortnight, shown as a percentage.

Teachers make use of this data in a variety of ways: when providing feedback, when designing and implementing interventions, when selecting small groups of students to work with, and when writing reports. Teachers are able to access precise entry- and exit-point data for each student against each topic, against the standards in the Australian Curriculum. Students can also be given a percentage grade for each topic based on their exit point, their rate of growth through the unit, or a combination of the two. Teachers are able to report in the same way they have always done, but many choose to shift the emphasis of their reports away from "level" and towards "growth".

Maths Pathway as a pedagogical model

The workflow described in the previous section certainly satisfies the requirement for practicable, manageable differentiated learning. However, the workflow itself is woven from different pedagogical elements that have been individually shown to be highly effective.

Part 1 – individual coursework

A large proportion of student time is spent on individual coursework. The form and flow of this coursework is specifically designed to build hand-written maths skills, keep students in the Zone of Proximal Development, and allow for constructivist learning.

Hand-written maths skills

The coursework completed by individual students is primarily hand-written. This is a purposeful choice.

Computers are used to collect, manage and analyse student data, as well as to guide student work as much as possible. There is a temptation for Maths Pathway to go further and fully computerise all student work; this is done by most online maths learning systems. Even more student data would be collected, and student work could be guided with even more precision. This would be entirely appropriate for a supplementary online resource that is used only occasionally in class, but not for a core tool like Maths Pathway. There are two main reasons for this.

The first is that hand-written mathematical skills *are important*. Even mathematicians, physicist and engineers – who use computers to do maths every day – still use hand-writing to organise thoughts, solve problems and do the thinking required to set up a computerised problem (Seto, 2006). Such written techniques go far beyond churning through computational algorithms; hand-writing in maths can give shape and structure to a problem, with the paper/whiteboard holding the information you can't hold in your head. Writing maths legibly also has proven correlations with overall achievement in mathematics (Oche, 2014). With Maths Pathway, hand-written skills are taught and assessed.

The second reason for retaining the hand-written component is to build students' independent learning skills. Maths Pathway does not give students only one online question at a time, but rather a set of connected questions with worked solutions. The expectation is for students to complete each question in sequence in their exercise books, and check their answers as they go. However, the reality is that students are able to make choices as they go: which questions to do and which ones to skip; how much working out to show in their books; whether to look at the answer first before doing the question; what summary notes to take as they go through. Students are trained by their teachers to make *good* choices as learners through the feedback cycle. The metacognitive and life-long learning skills students develop are invaluable (Issa, Issa, & Kommers, 2014, p. 29).

Zone of Proximal Development

Individual students' coursework is always within Vygotsky's Zone of Proximal Development (ZPD) (Morgan, 2013) That is, the work is too difficult for students to complete automatically, but just within their reach with the right scaffolding. This feature of Maths Pathway is vital for the development of conceptual understanding, as has already been discussed. However, there are also psychological benefits to this approach that go beyond the pure pedagogy – relating to Mathematics Anxiety and self-efficacy.

Mathematics Anxiety (Lions & Beilock, 2012) is a well-documented phenomenon. A person's experience of repeated failure in mathematics manifests in a fear response whenever a mathematical problem is presented. The fear response itself actually impairs the ability to solve problems (Lions & Beilock, 2012), and so for a student the problem can compound over time. However, if a student is held constantly within the ZPD, there is a constant chance for success. This gives the teacher a genuine chance to overcome the initial barrier Mathematics Anxiety creates, and form new habits and experiences of success with students. One common student comment teachers receive when using Maths Pathway is "I have a better understanding of maths and am finally starting to catch up".

The ZPD can also help develop students' self-efficacy (Ferguson, 2009). That is, the way they view themselves – their skills, capabilities and potential. Our culture views mathematics in a somewhat paradoxical way: people will boast of their ignorance of mathematics (Burns, 1998) but at the same time regard anyone who *can* do mathematics as being extremely clever – even as a genius (Andrews, 2011). This can translate into the way students view or classify themselves. Student self-perception around intelligence or even overall academic prowess is often tied to their success in mathematics. The ZPD allows all students to be successful in mathematics, and therefore enhances their self-efficacy. This may be why many students comment "I feel smarter" when teachers survey them about their Maths Pathway experience.

Constructivism

In constructivist learning (Poncy, McCallum, & Schmitt, 2010), students build new knowledge from existing knowledge. By constructing their own knowledge wherever possible, student gain an intrinsic understanding of the way mathematical concepts connect together. This can be quite difficult to achieve in mathematics classrooms, and students are usually provided with direct instruction and explicit modelling of every mathematical concept prior to any practice work. By contrast, Maths Pathway coursework has a largely constructivist basis – via careful assignment of coursework, and also via the structure within each learning activity.

Maths Pathway's data and workflow ensures that specific prerequisites are met prior to any activity being set to a student. The activity's questions can safely assume mastery and deep understanding in the prior knowledge. Generally, the first few questions in each learning activity provide a quick revision opportunity, and call to mind the key facts or concepts ready to be drawn upon. This sets students up to be able to derive or experiment with whatever *new* concept or skill they are now aiming to acquire.

Beyond this, careful scaffolding (Wyatt-Smith, Klenowski, & Colbert, 2014, pp. 83-85) is needed throughout the whole learning activity to ensure that students can build their own understanding. Scaffolding for eventual success with assessment hinges on scaffolding for understanding. Each question builds on the last, introducing key elements of thinking a bit at a time. For instance, there might be a need to model a process, so an incomplete process is given to the students and their job is to fill in the blanks. A particular misconception might need to be targeted, so students are given a sample of incorrect student work and have to find out what is wrong with it. Practice of a particular process or skill may be needed, so this too is broken down into different cases for students to tackle individually, and then connect together. Often this scaffolding structure is spread over multiple activities, as well as being present within each activity itself. It is for this reason that Maths Pathway provides a complete set of learning activities to teachers; using existing text book resources or worksheets would not allow constructivist learning to take place.

Part 2 – fortnightly feedback cycle

Assessment and feedback take place over fortnightly cycles. The assessments themselves include assessment for learning, as learning and of learning. Feedback is provided with the utmost care and attention. Student success is actively framed so as to foster the development of a growth mindset.

Assessment for learning, as learning, of learning

Assessment for learning, assessment as learning and assessment of learning (Board of Studies New South Wales, 2015) are all woven into the fortnightly feedback cycle.

In assessment as learning, students learn to self-assess and self-diagnose. This is why students co-construct their own maths tests within Maths Pathway, by choosing what they want to be tested on. The composition of the test at the end of each fortnight is determined by the choices that students make leading up to it. Students only complete each activity once they say they are ready for assessment. Students also revisit each assessable activity during test revision. Throughout, students have the choice to do extra practice or ask for assistance if needed. A large part of the feedback teachers provide is centred on the metacognitive choices students have made; good and bad.

In assessment of learning, teachers use "evidence of student learning to assess achievement against outcomes and standards" (Board of Studies New South Wales, 2015). Every assessment in Maths Pathway provides assessment of learning, including the diagnostic assessments that take place at the start of each unit and the formal tests that occur each fortnight. Every assessment item and learning activity is built explicitly around the Australian Curriculum standards and mapped against them, so that teachers always have up-to-date information about student capabilities, levels and growth. The benefits of regular reporting and feedback go hand-in-hand with the power of differentiation (Wyatt-Smith, Klenowski, & Colbert, 2014). It is very easy for teachers to report using Maths Pathway data, whether this is once a term or on an ongoing basis as with continuous reporting.

In assessment for learning, the data obtained from assessments are used to inform the teaching that follows. Every piece of Maths Pathway assessment is of this type. Primarily, this is because all data collected on student learning updates their individual learning paths, which in turn determines what learning outcomes students are working towards week by week. However, teachers also use this data to identify which students to group together for targeted direct instruction, and which individuals to intervene with when needed. This may be based on the student's current capabilities, but also on their rate of growth, their work completion or a wide variety of other metrics available to teachers.

Feedback

Providing students with feedback is known to be the most effective thing a teacher can do to impact learning outcomes (Voerman, Meijer, Korthagen, & Simons, 2012). The entire Maths Pathway workflow has feedback as a cornerstone. This feedback is of high quality, is meaningful to the student, and is also practical for the teacher to implement.

The high quality of feedback within Maths Pathway comes from how specific and personal it is. Data from students' guided self-reflection allows teachers to distinguish small errors from genuine misconceptions after each assessment. As the teacher and student go through their feedback interview, the teacher has evidence for the important choices the student made during the prior fortnight, and also during the test. If a student has made a minor error, teachers often talk with them about proof-reading their answers or re-reading questions. If a student has forgotten something, teachers often talk about summarisation and revision strategies. If a student is genuinely stuck on something, teachers often talk about the student's decision to move on from a learning activity prematurely, and about what help the student sought – even about how the student is using worked solutions or organising their bookwork. Some of the most effective feedback teachers have provided to date have focussed explicitly on soft skills (Heckman & Kautz, 2012) and habits (Duhigg, 2012).

Feedback within Maths Pathway is always meaningful and relevant to students. In traditional classroom workflows, when students receive feedback from an assessment task they often look only at their grade. The question-by-question feedback is where most of the value lies, but this is often ignored by the student as irrelevant; after all, the class is now moving onto another topic. This is why the Maths Pathway workflow repeats assessment. If a student makes a mistake on a test question, they know that their next test will contain a very similar question so they can try again. Students view feedback as a useful part of their learning experience – something that will genuinely help them to perform better next time. Maths Pathway teachers often remark that the lessons where reflection and feedback take place are ones in which a huge amount of learning happens, because students are so invested in getting it right the *next* time.

Arguably the most important aspect of Maths Pathway feedback is that it's actually practical. Feedback is something which teachers never seem to have enough time to do as well as they would like. A model in which every student has a one-on-one interview with the teacher after each assessment is all very well, but only if the teacher actually has the time during class to do this. The Maths Pathway workflow is built around this constraint. Once a fortnight after each assessment, teachers spend one or two lessons working through feedback interviews with individuals while the rest of the class continues with their coursework. They are able to use data to identify which students require feedback most urgently, but almost every student receives meaningful feedback every fortnight. This requires no extra preparation by teachers outside of class, and the interviews are conducted during class time; not at lunch or recess.

Growth Rate as a metric of success

When students adopt a growth mindset (Dweck, 2006), they perceive intelligence as being not fixed and pre-determined, but something that can be developed and worked on. Maths Pathway helps students to see that they are in control over their own success in mathematics, and communicates that success to students explicitly in terms of growth.

It is natural for students to gauge their aptitude for mathematics by the degree of success that they experience in maths relative to their peers. In traditional one-size-fits-all classroom models, some students experience success with coursework far more easily than others simply by virtue of entering at a higher level. Maths Pathway provides a more level playing field: the degree of personal challenge is the same for every student in the class. Success becomes less about who you are and more about how you are working. Students are therefore able to genuinely perceive their mathematical aptitude as being in within their control, which is a vital element of a growth mindset.

In Maths Pathway, the primary metric used to communicate student success is "growth rate". This is specifically to foster the development of a growth mindset. A student gets 100% on a test is they master two weeks' worth of new mathematics within that fortnight. This is irrespective of the student's entry point – it is the growth and learning that is rewarded most heavily, not the overall level. Students still have an awareness of their overall level in maths, but it is transparent that they have control over their rate of growth, and that by extension they have control of their level. It is possible for teachers to frame success for students in such terms without Maths Pathway, but this way of measuring success adds weight and relevance to the narrative and therefore attains a much higher degree of student buy-in.

Part 3 – collaborative work

Individualised work can be highly effective at developing students' skills and understanding, but collaborative activities are also a vital part of the learning experience. Participation in rich tasks and meaningful discourse about mathematics are both key ingredients in the Maths Pathway learning model. Teachers use at least one lesson per week targeting rich, collaborative learning.

Rich tasks

Rich tasks provide great opportunities for kinaesthetic learning (Steele, 2013). These often provide an entry-point for every student in the class, allowing all students the opportunity to participate in group problem-solving activities. Moreover, they help to form different types of connections in the brain, and can help deepen or develop students' understanding of mathematical concepts.

Non-routine problem-solving is also a key element within rich learning activities. Such problems often require students to work in groups to apply whatever mathematical tools they have at their disposal in a new and unfamiliar way. Such application is not only a valuable skill in and of itself, but also forms higher-level connections between different areas of mathematics. The process of abstraction from a real-world problem to a mathematically formulated one is absolutely key for students in attempting any sort of worded problem they come across in later studies, or in any application of maths in later life.

Discourse

Learning to communicate mathematical ideas is a key part of what students need to learn (Güçler, 2012). During individualised work, students learn to communicate via writing and pictures, but it is collaborative work that provides opportunity for verbal and aural communication. Discourse between students during a group-based activity gives students an opportunity to form their own mathematical ideas into sentences, challenge the thinking and logic of their peers, and understand a piece of mathematics from another person's point-of-view.

Discourse between students and teachers during collaborative learning activities is also pivotal. Students attempt to verbalise their thinking, conjectures and logic, which provides invaluable opportunity for the teacher to engage in critical questioning. This reciprocal learning approach can aid in disequilibriation of ineffective rote learning practices (Sutherland, 2006). Via scaffolding general problem-solving strategies within rich tasks, which demand the engaging variety of more than one way of thinking about the solution to a problem (Steele, 2013), often triggering those "ah-ha moments" in students. This teacher-student dialogue can involve individual students, groups of students or the class as a whole.

Part 4 – student motivation

The inculcation of intrinsic motivation for students is practically a universal priority for teachers, regardless of the subject they teach. In mathematics, it is often difficult for students to see their work as meaningful (Martin, 2013); that is, to have autonomy, mastery and purpose in their maths learning. The Maths Pathway workflow makes each of these far easier to achieve. The maths work being meaningful is what provides the chief motivation drive for students (Pink, 2009).

Autonomy

One ingredient for meaningful work is autonomy. That is, to have an element of choice and agency over the work you are doing. This is why the Maths Pathway workflow allows student a “longer leash” than usual for two weeks at a time, during which students are free to make both good and bad choices as learners. These choices include how much work to do, how to utilise worked solutions, which questions to attempt or skip, when to ask for help, and how to prepare for assessments. There is sufficient guidance that students are in a position to make good choices most of the time, and sufficient accountability through feedback to help students get better at making choices. Student autonomy is a key part of the experience.

Mastery

The next part of meaningful work is mastery. That is, feeling as though you are getting better at something as you go along. To a certain extent, this happens naturally in mathematics; each year students are capable of attempting maths that would have been incomprehensible to them the year before. Maths Pathway helps to make this even more apparent and visual to students at the end of each fortnight. Students see their mastery increasing on a pictorial representation of the learning map, and also couch all of their success in mathematics through growth – the rate of acquisition of new mastery.

Purpose

The last thing required for meaningful work is a sense of purpose. That is, knowing that you are working towards something meaningful. In the Maths Pathway workflow, rich learning is absolutely key for this. Students are able to apply their ever-growing mathematical toolkit to interesting and real-world problems. It becomes entirely transparent to students how mathematical thinking enables them to perform valuable and complex tasks, and that they become better at this over time. Rich learning not only connects all of mathematics together, but makes it purposeful.

All of the above adds up to a shift towards more intrinsic motivation within students; an enjoyment of mathematics for its own sake. This is not the result of gamification, or artificial-feeling ties to the real world, but stems from a genuine sense of students’ work in mathematics class as being meaningful.

References

Andrews, B. (2011). What happened to science education? *Astronomy, 39*(8), 45. Retrieved from http://www.astronomy.com/~/media/Files/PDF/Magazine%20articles/science-education.ashx

Arslan, S. (2010). Traditional instruction of differential equations and conceptual learning. *Teaching Mathematics and its Application, 29*(2), 94-107. doi:10.1093/teamat/hrq001

Australian Curriculum Assessment and Reporting Authority. (2015). *Mathematics Rationale*. Retrieved February 5, 2015, from http://www.australiancurriculum.edu.au/mathematics/rationale

Australian Curriculum Assessment and Reporting Authority. (2015). *Student Diversity*. Retrieved February 5, 2015, from http://www.australiancurriculum.edu.au/studentdiversity/meeting-diverse-learning-needs

Barr, C., Doyle, M., Clifford, J., De Leo, T., & Dubeau, C. (2003). *There is More to Math: A Framework for Learning and Math Instruction*. Retrieved February 6, 2015, from http://teachingmathliteracy.weebly.com/conceptual-vs-procedural-knowledge.html

Board of Studies New South Wales. (2015). *Assessment for, as and of Learning*. Retrieved February 5, 2015, from http://syllabus.bos.nsw.edu.au/support-materials/assessment-for-as-and-of-learning

Burns, M. (1998). *Math: Facing an American Phobia.* Minneapolis: Maths Solutions Publications.

Cobb, P. (2010). The tension between theories of learning and instruction in mathematics education. E*ducational Psychologist, 23*(2), 87-103. doi:10.1207/s15326985ep2302_2

Department of Education and Training. (2013). *Mandated Components of the Report Card.* Retrieved February 5, 2015, from http://www.education.vic.gov.au/schools/teachers/support/Pages/mandatedcomponents.aspx

Duhigg, C. (2012). *The Power of Habit: Why We Do What We Do in Life and Business.* New York: Random House.

Dweck, C. (2006). *Mindset: The new psychology of success.* New York: Random House.

Farr, S. (2010). *Teaching as Leadership: The highly effective teacher's guide to closing the achievement gap.* San Francisco: Teach For America; A Wiley Imprint.

Ferguson, S. (2009). Same task, different paths: catering for student diversity in the mathematics classroom. *Australian Primary Mathematics Classroom, 14*(2), 32-36.

Forgasz, H. (2010). Streaming for mathematics in years 7-10 in Victoria: an issue of equity?. *Mathematics Education Research Journal, 22*(1), 57-90. doi:10.1007/BF03217559

Güçler, B. (2012). Limitless Ways to Talk about Limits: Communicating Mathematical Ideas in the Classroom. *The Mathematics Teacher, 105*(9), 697-701.

Hattie, J. (2009). *Visible Learning: A synthesis of over 800 Meta-Analyses Relating to Achievement.* New York: Routledge.

Heckman, J., & Kautz, T. (2012). Hard evidence on soft skills. *Labour Economics 19*(4), 451-464. doi:10.3386/w18121

Issa, T., Issa, T., & Kommers, P. (2014). *Feedback and Learning Support that Fosters Students' Independent Learning: An Australian Case Study*. Netherlands: University of Twente.

Jones, H., & Russell, J. (2006). Hierarchical learning paradigm. *Journal of Research in Science Teaching, 16*(6) 489-499. doi:10.1002/tea.3660160603

Juan, A., Huertas, M., Trenholm, S., Steegman, C. (2012). *Teaching Mathematics Online, Emergent Technologies and Methodologies: Long-Term Experiences in Mathematics E-Learning in Europe and the USA.* Pennsylvania: Hershey

Lions, I., & Beilock, S. (2012). Mathematics anxiety: separating the math from the anxiety. *Cerebral Cortex, 22*(9), 2102-2110. doi:10.1093/cercor/bhr289

Martin, J. (2013). Handing Over the Keys: Giving Students Ownership of Math. *Middle Ground, 16*(3), 22.

Morgan, H. (2013). Maximising Student Success with Differentiated Learning. *The Clearing House: A Journal of Educational Strategies, Issues and Ideas, 87*(1), 34-38. doi:10.1080/00098655.2013.832130

Oche, E. (2014). The influence of poor handwriting on students' score reliability in mathematics. *Mathematics Education Trends and Research, 2014,* 1-15. doi:10.5899/2014/metr-00035

Pink, D. (2009). *Drive: The Surprising Truth about What Motivates Us*. New York: Riverhead Books.

Poncy, B., McCallum, E., & Schmitt, A. (2010). A comparison of behavioural and constructivist interventions for increasing math-fact fluency in a second grade classroom. *Psychology in the Schools, 47*(9), 917-930.

Setto, B. (2006). Writing in mathematics: making it work. *Primus, 16*(3), 204-232.

Steele, M. (2013). Exploring the mathematical knowledge for teaching geometry and measurement through the design of rich assessment tasks. *Journal of Mathematics Teacher Education*, *16*(4), 241.

Sutherland, R. (2006). *Teaching for Learning Mathematics: Chapter 5 Teaching and Learning as Reciprocal Activity.* New York: McGraw Hill Education.

Voerman, L., Meijer, P., Korthagen, F., & Simons, R. (2012). Types and frequencies of feedback interventions in classroom interaction in secondary education*. Teaching and Teacher Education*, *28*(8), 1107-1115.

Wyatt-Smith, C., Klenowski, V., & Colbert, P. (2014). *Designing Assessment for Quality Learning.* Queensland: Springer.